AP Calculus BC

Part I
Summer Pre-View Packet

Suggested Due Date: Monday, August 5th by 12pm in Mrs. Garrido’s mailbox in the main office at RFH.
(no points will be deducted for handing it in with part 2 on the first day of school. It is in your best interest to complete this half of the assignment by this date to give you time to complete the second half of the summer assignment)

Questions?
Contact
Mrs. Garrido: lgarrido@rumsonfairhaven.org

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Working with a tutor is not acceptable.
Part I: Please show all work on this worksheet for each question to receive full credit. Calculators are not permitted.

Find the equation for the specified line in point-slope form: \( y = m(x - x_1) + y_1 \)

1. through (1, -3) and (-4, 6)

2. through (-2, -3) and perpendicular to \( 3x - 9y = 12 \)

Complete the following for #s 3 – 8

a) Draw an accurate graph of the function
b) Identify the domain and range
c) Identify the end behavior

3. \( f(x) = 2|x+3|-1 \)  
   Domain: ____________  Range: ____________  E.B.: ____________

4. \( y = \frac{1}{x} \)  
   Domain: ____________  Range: ____________  E.B.: ____________

5. \( y = e^x \)  
   Domain: ____________  Range: ____________  E.B.: ____________
6. \( y = \ln(x) \)

7. \( y = \sqrt{x - 3} \)

8. \( y = \frac{1}{x^2 + 2} \)

Domain: ____________  Domain: ____________  Domain: ____________

Range: ____________  Range: ____________  Range: ____________


Complete the following for #s 9-11:

a) Simplify
b) Identify holes
c) Identify the zeros
d) Identify vertical asymptotes
e) Identify horizontal / slant asymptote
f) Sketch the rational function
g) Identify domain and range
h) Identify end behavior

9. \( f(x) = \frac{x^3 - x}{x^2 - 4} \)
10. \( f(x) = \frac{x - 4}{x^2 - 3x - 4} \)

11. \( f(x) = \frac{x^2 - 4x - 32}{x^2 - 16} \)
Complete the following for #s 12-14
a) Tell whether the functions are even, odd, or neither
b) Explain using function notation
c) Identify any lines of symmetry. (y-axis, origin, neither)

12. \( y = \sin(x) \)

13. \( y = \cos(x) \)

14. \( f(x) = \sqrt{x^2 - 3} \)

Graph the following piecewise function. Be sure to draw accurate graphs and include a scale.

15. \[
\begin{cases} 
\sin x + 1, & -2\pi < x < 0 \\
2\cos x, & 0 \leq x < \pi 
\end{cases}
\]
16. Find the following limits based on the graph below.

\[ \lim_{x \to 0^-} x = \]

\[ \lim_{x \to 0^+} x = \]

\[ \lim_{x \to 0} x = \]

Find the following limits algebraically.

17. \[ \lim_{x \to 3} (x^2 - 5x + 3) = \]

18. \[ \lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4} = \]
19. Give the simplified value/equation for the following 8 questions based upon the function notation:

\[
\text{If } \begin{align*}
    f(x) &= \{(3,5), (2,4), (1,7)\} \\
    g(x) &= \sqrt{x-3} \\
    h(x) &= \{(3,2), (4,3), (1,6)\} \\
    k(x) &= x^2 + 5
\end{align*}
\]

determine each of the following:

1. \((f + h)(1) = \)

2. \((k - g)(5) = \)

3. \(f(h(3)) = \)

4. \(g(k(7)) = \)

5. \(f^{-1}(x) = \)

6. \(k^{-1}(x) = \)

7. \(\frac{1}{f(x)} = \)

8. \((kg)(x) = \)
For #20-25, evaluate the following trig functions.

20. \( \sin \left( \frac{2\pi}{3} \right) = \) __________  
21. \( \cos \left( \frac{5\pi}{2} \right) = \) __________

22. \( \tan \left( -\frac{3\pi}{4} \right) = \) __________  
23. \( \csc \left( -\frac{11\pi}{6} \right) = \) __________

24. \( \sec ( \pi ) = \) __________  
25. \( \cot ( -\pi ) = \) __________

For #26-29, find the **exact** (not decimal) value of \( x \).

26. \( (1.23)^x = 4 \)  
27. \( e^{\ln(x)} = 20.03 \)

28. \( \ln x = 6 \)  
29. \( \log_2 5 - \log_2 x = 3 \)
For #30-32, simplify completely:

30. Eliminate the time parameter and write a rectangular equation for:
    
    \[ x = t^2 + 3 \quad \text{and} \quad y = 2t \]

    \[ y = \text{____________} \]

31. \[ \frac{2}{5} \cdot \frac{4}{9} = \text{____________} \]

32. \[ \frac{3(n + 1)!}{5n!} = \text{____________} \]

For #33-35, find all real solutions of \( x \):

33. \[ 2x^2 + 13x - 24 = 0 \]
34. \( x + \frac{4}{x} = 4 \)

35. \( \sqrt{2x - 3} - 2 = 0 \)

Determine all points of intersection of the two graphs:

36. Parabola \( y = x^2 + 3x - 4 \) and line \( y = 5x + 11 \)
**Inverse Functions**

37. Find the inverse function of \( y = 3x^3 + 2 \), if it exists.

38. Given the vectors \( \mathbf{v} = -2\mathbf{i} + 5\mathbf{j} \) and \( \mathbf{w} = 3\mathbf{i} + 4\mathbf{j} \), determine:

   - \( \frac{1}{2} \mathbf{v} \)
   - \( \mathbf{w} - \mathbf{v} \)
   - The length of \( \mathbf{w} \)
   - The unit vector for \( \mathbf{v} \)
39. For parts a-d, complete the following:
- Identify, by name, each polar graph.
- Give at least one characteristic of each graph (ex. Radius, location, length of petal, point (other than the pole) on the graph, etc.)
- Sketch a graph of each

- **a) \( r = 2 \)**
  ![Graph of \( r = 2 \)]

- **b) \( r = 3 \sec \theta \)**
  ![Graph of \( r = 3 \sec \theta \)]

- **c) \( r = 1 + \sin \theta \)**
  ![Graph of \( r = 1 + \sin \theta \)]

- **d) \( r = 2 \cos 3 \theta \)**
  ![Graph of \( r = 2 \cos 3 \theta \)]
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Part II: Show all work on this worksheet when necessary for full credit.
A calculator is permitted on this section.

1. Given \( g(x) = x^3 - 2x^2 - 2x + 3 \) over the interval \([-3, 1]\), use your graphing calculator to find:

   a) the zeros of \( g \)

   b) the minimum value of \( g \) and where it occurs

   c) the maximum value of \( g \) and where it occurs

   d) the \( y \)-intercept of \( g \)

   e) the \( x \)-intercepts of \( g \)

Geometry

2. Find the volume and surface area of the following (include units):

   a. Sphere with diameter 6 mm

      Volume =

      S.A. =

   b. Cylinder with radius 3 in. and height 7 in

      Volume =

      S.A. =
c. Cone with diameter 16 ft. and height 15 in.

\text{Volume} =

\text{S.A.} =

d. Cube with edge 6m.

\text{Volume} =

\text{S.A.} =

3. Find the following for the line segment \( AB \) if \( A = (6, -8) \) and \( B = (9, 18) \).

a. Slope = ______

b. Length = ______

c. Midpoint = ______
4. Evaluate \( \frac{f(x + h) - f(x)}{h} \) and simplify if \( f(x) = x^2 - 2x \).

Applications

Growth / Decay:

5. The weight of a radioactive substance after \( t \) years is given by \( W = 1500 \times (0.993)^t \) grams. Find the following:

   a. the original amount of radioactive material

   b. the amount of radioactive material remaining after 400 years

   c. how long it would take to reduce the weight to 100 grams.
6. The population of water buffalo is given by \( P(t) = 400 + 250 \sin(90t) \) where \( t \) is the number of years since the first estimate was made.

a. What was the initial population?

b. What was the population size after 6 months?

c. Use technology to find the first time interval when the herd exceeds 500 (use degree mode).

7. A manufacturer finds it costs him \( x^2 + 5x + 7 \) dollars to produce \( x \) tons of an item. At production levels above 3 tons, he must hire additional workers, and his costs increase by \( 3(x - 3) \) dollars on his total production. If the price he receives is $13 per ton regardless of how much he manufactures and if he has a plant capacity of 10 tons, what level of output maximizes profit?
8. Consider the circle \( x^2 + y^2 - 6x - 8y = 0 \)

   a. Find the center and radius of the circle

   b. Find an equation of the tangent line to the circle at the point \((0,0)\)

   c. Find an equation of the tangent line to the circle at the point \((6, 0)\)

   d. Where do the two tangent lines intersect?
9. A man has 340 yards of fencing for enclosing two separate fields, one of which is to be a rectangle twice as long as it is wide and the other a square. The square field must contain at least 100 square yards and the rectangular one must contain at least 800 square yards.

   a. If \( x \) is the width of the rectangular field, what are the maximum and minimum possible values of \( x \)? (Must write equations used)

   b. What is the greatest number of square yards that can be enclosed in the two fields? Justify your answer. (Must write equations used)

10. A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs $10 per square meter for the base and $5 per square meter for the sides, what is the cost of the least expensive tank? (Must write equations used)
11. Find the area of the largest rectangle (with sides parallel to the coordinate axes) that can be inscribed in the region enclosed by the graphs of \( f(x) = 18 - x^2 \) and \( g(x) = 2x^2 - 9 \).